**SDI Spotlight – Mathematics: Whole Number Operations**

**SDI Spotlight Purpose:** This spotlight was based on practices identified in VDOE’s [Evidence Based Specially Designed Instruction in Mathematics](https://vdoe.prod.govaccess.org/home/showdocument?id=3206), and focuses on specific strategies for whole number operations using the Concrete-Representational-Abstract (CRA) method, an evidence-based practice. Brief videos demonstrating how to teach whole number operations with CRA and lesson plans when available are shared.

**Considerations:** Dr. Sarah Powell, Assistant Professor discusses important considerations when teaching students with difficulties in mathematics (8:22 mins): [Considerations for teaching students with math difficulties video](https://youtu.be/dQX9Cl0s04I). To better understand the needs of students with mathematics disabilities, read: [VDOE Students with Disabilities in Mathematics Frequently Asked Questions](https://vdoe.prod.govaccess.org/home/showdocument?id=3204).

**Explicit Instruction (HLP 16):** Explicit instruction forms the foundation for delivering specially designed instruction. CEC created a checklist that walks teachers through phases of explicit instruction: [CEC HLP 16 Checklist](https://ttaconline.org/Resource/JWHaEa5BS74Th4roZsxqhg/Resource-hlp-16-checklist-explicit-instruction-high-leverage-practices-implementation-guide).

**Proper Mathematics Vocabulary:** It’s essential to use and encourage student dialogue with proper mathematical vocabulary. VDOE identified important mathematics vocabulary: [VDOE Word Wall Cards](https://www.doe.virginia.gov/teaching-learning-assessment/k-12-standards-instruction/mathematics/instructional-resources/mathematics-vocabulary-word-wall-cards). Scaffolds like [Frayer Models](https://iris.peabody.vanderbilt.edu/module/sec-rdng/cresource/q2/p07/) enhance students’ understanding of mathematics vocabulary. For students with decoding difficulties, decoding strategies and syllabication can integrate with vocabulary word instruction. This [video from Anita Archer](https://explicitinstruction.org/video-elementary/elementary-video-4/) demonstrates how to introduce new vocabulary words with decoding strategies.

**Progress Monitoring:** Monitoring student progress is an essential component of instruction. When students are not making meaningful progress, we gather data to analyze instructional practices and make necessary adjustments to improve student outcomes. Progress Monitoring Tools:

* National Center on Intensive Interventions [Student Progress Monitoring Tool for Data Collection and Graphing](https://intensiveintervention.org/resource/student-progress-monitoring-tool-data-collection-and-graphing-excel)
* Adapted Virginia Tech TTAC [CRA Mathematics Progress Monitoring Data Collection Form](https://gmuedu-my.sharepoint.com/%3Aw%3A/g/personal/cmarti82_gmu_edu/Ecxfdxr7JUFAgaMk15BTo4IBoD5XLkj3WTuT-mqVf49xWA?e=LdjcXt)

**Learn:** How to interpret progress monitoring data: [Project Stair (4:29 mins.)](https://www.youtube.com/watch?v=O3IPT5fX6YY) and how to use error analysis in mathematics [IRIS Center (2015) Page 7 Error Analysis for Mathematics](https://iris.peabody.vanderbilt.edu/module/dbi2/cresource/q2/p07/).

## Concrete-Representational Abstract (CRA) Method

CRA is an evidence-based practice. Listed below, are resources to learn more:

* Read about CRA & Visual Representations [IRIS Center (2017) Page 5 Visual Representations](https://iris.peabody.vanderbilt.edu/module/math/cresource/q2/p05/).
* VDOE’s [Evidence Based Specially Designed Instruction in Mathematics](https://vdoe.prod.govaccess.org/home/showdocument?id=3206)
* Learn about CRA: [LD@School self-paced learning - Concrete-representational-abstract method](https://www.ldatschool.ca/learning-modules/cra-strategies/overview/).

**Virtual Manipulatives (free)**

* [Didax Virtual Manipulatives](https://www.didax.com/math/virtual-manipulatives.html)
* [EquatIO Activities Database](https://mautic.texthelp.com/equatio-activity-database)
* [Kentucky Center for Mathematics](https://www.kentuckymathematics.org/vr_other.php)
* [Math Playground](https://www.mathplayground.com/)
* [Math Learning Center](https://www.mathlearningcenter.org/apps)
* [National Library of Virtual Manipulatives](http://nlvm.usu.edu/en/nav/vlibrary.html)
* [Toy Theatre](https://toytheater.com/category/teacher-tools/virtual-manipulatives/)

### What does the CRA Method for Whole Numbers Look Like?

Listed below are specific research-based interventions/practices to teach whole number operations with CRA. Interventions/practices are listed by whole number property (addition, subtraction, multiplication, division). When available, lesson plans and videos area shared. For comprehensive professional learning: Check out [National Center on Intensive Intervention (Module 6 Whole Number Operations)](https://intensiveintervention.org/whole-number-math-course)

| **Whole Number Property** | **Intensive Intervention Strategy** | ***Some* Lesson Plans, Videos & Activities** |
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| AdditionNational Center of Intensive Intervention [Instructional Video of Part-Part-Whole & Join Model](https://www.youtube.com/watch?v=RMRnt1yJ4tw)(1 minute) | Part-Part-Whole* Initial way to teach addition.

Join Model* More advanced way to teach addition.
* Start with a set and add or join another set.

Teach Commutative Property of Addition | Part- Part-Whole Lesson Plan* [TeachingHub](https://www.teachhub.com/in-the-classroom/lesson-plans/2020/07/part-part-whole-relationships/)

[National Center on Intensive Intervention’s Sample Mathematics Lessons for Intensifying Interventions](https://intensiveintervention.org/implementation-intervention/math-lessons)[Khan Academy Number Line Addition Video & Activities](https://www.khanacademy.org/math/arithmetic-home/addition-subtraction/basic-add-subtract/v/basic-addition)[CRA with Integers Lesson Plan from Learning Disabilities Association](https://www.ldatschool.ca/wp-content/uploads/2014/07/CRA_integers_final_accessible1.pdf)Henrico County, VA [Mathematics Courses – Activities and links that are aligned to grade level SOL](https://sites.google.com/henrico.k12.va.us/hcpsmathematics/courses?authuser=0) |
| SubtractionNational Center on Intensive Intervention [Subtraction with Manipulatives Video](https://www.youtube.com/watch?v=vR124wQWhj8)(1 minute) | Separate* Change decrease.
* Compare
* Difference Problems
 | [National Center on Intensive Intervention’s Sample Mathematics Lessons for Intensifying Interventions](https://intensiveintervention.org/implementation-intervention/math-lessons)[Khan Academy Subtraction with Number line video and activities](https://www.khanacademy.org/math/arithmetic-home/addition-subtraction/basic-add-subtract/v/basic-subtraction) Khan Academy [Commutative Property of Addition with manipulatives](https://www.khanacademy.org/math/arithmetic-home/multiply-divide/properties-of-multiplication/a/commutative-property-review#:~:text=What%20is%20the%20commutative%20property,does%20not%20change%20the%20product.) Henrico County, VA [Mathematics Courses – Activities and links that are aligned to grade level SOL](https://sites.google.com/henrico.k12.va.us/hcpsmathematics/courses?authuser=0) |
| MultiplicationNational Center on Intensive Intervention [Equal Groups Demonstration Video](https://www.youtube.com/watch?v=0FNNiJVHFRs) (1 minute) | Equal groups * *example: 3 x 2 is 3 groups of 2*
* Initial instruction

Equal groups – ArrayCompare* More advanced
* Number line (show arrow to negative infinite, include 0)
 | Henrico County, VA [Mathematics Courses – Activities and links that are aligned to grade level SOL](https://sites.google.com/henrico.k12.va.us/hcpsmathematics/courses?authuser=0)[National Center on Intensive Intervention’s Sample Mathematics Lessons for Intensifying Interventions](https://intensiveintervention.org/implementation-intervention/math-lessons)[Khan Academy – Equal Groups](https://www.khanacademy.org/math/cc-third-grade-math/intro-to-multiplication/imp-multiplication-intro/v/skip-counting-equal-groups)Utah Education Network – [Multiplication Using Arrays Lesson Plan](https://emedia.uen.org/courseware/lesson/1232/overview)Betterlessons.com - [Equal groups on a number line lesson plan](https://teaching.betterlesson.com/lesson/524781/equal-groups-on-a-number-line) Henrico County, VA [Mathematics Courses – Activities and links that are aligned to grade level SOL](https://sites.google.com/henrico.k12.va.us/hcpsmathematics/courses?authuser=0) |
| DivisionNational Center on Intensive Intervention [Division with Concrete Manipulatives Video](https://www.youtube.com/watch?v=D4RU1TCvIHA)(1 minute) | Partitive (Equal Shares Model)* Show dividend.
* Divide equally among divisor (groups)
* Count to get quotient.
* Measurement
 | Henrico County, VA [Mathematics Courses – Activities and links that are aligned to grade level SOL](https://sites.google.com/henrico.k12.va.us/hcpsmathematics/courses?authuser=0)[National Center on Intensive Intervention’s Sample Mathematics Lessons for Intensifying Interventions](https://intensiveintervention.org/implementation-intervention/math-lessons) |

### Advanced Whole Number Operations

**Vocabulary:** Regrouping (exchanging, trading) – known years ago as “carry-over,” which is not proper mathematics vocabulary.

* If an answer is more than 9, have to regroup.
* Regroup one **10** for **ten** **1**’s

**Visual-Spatial Organization Strategies for Place Value:**

Many students with disabilities have visual spatial issues that adversely impact their ability to properly align numbers. The following strategies help many students with number alignment to organize place value.

* Graph paper, Notebook paper sideways, Bookmark

### What Does It Look Like?

#### Addition:

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| **Standard Algorithm** * Right to left addition.
* Regrouping

[Regrouping Video Demonstration](https://www.youtube.com/watch?v=VPsYRPdlIpU) | **Partial Sums Strategy Addition*** Left to right addition within place value.
* Sum each place value for partial sums
* Sum all partial sums

National Center on Intensive Intervention [Partial Sums Strategy Addition Video](https://www.youtube.com/watch?v=TClOCPBYw10) |

#### Subtraction:

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| **Standard Algorithm*** Right to left subtraction.
* Regrouping

National Center on Intensive Interventions [Standard Subtraction Video](https://www.youtube.com/watch?v=fL9H6VPsoi4) | **Subtraction – Partial Differences*** Uses integers.
* Subtract place values.
* Add differences.

National Center on Intensive Interventions [Partial Differences Video](https://www.youtube.com/watch?v=49cnkJBrS9k) | **Add Up Strategy*** Only addition
* Use number line to help visualize.
* Select an easy number.
* Calculate differences.
* Add differences.

[Add Up strategy video demonstration](https://www.youtube.com/watch?v=AiSKj8wI4rM) |

#### Multiplication:

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| **Standard**National Center on Intensive Intervention- [Traditional Algorithm video](https://youtu.be/85xxAfNP_vM)Khan Academy – [Multiplication Videos](https://www.khanacademy.org/math/arithmetic-home/multiply-divide/multi-digit-mult/v/2-digit-times-1-digit-example-no-carrying) | **Equal Groups*** Show the groups.
* Multiplication sign means “groups of”
* Show the amount within each group
* Count the product

Khan Academy – [Equal Groups](https://www.khanacademy.org/math/cc-third-grade-math/intro-to-multiplication/imp-multiplication-intro/v/skip-counting-equal-groups)National Center on Intensive Interventions [Equal Groups](https://www.youtube.com/watch?v=D4RU1TCvIHA)National Center on Intensive Interventions[Using base 10 blocks to demonstrate multiplication](https://youtu.be/RTv6yt2iHJs)National Center on Intensive Intervention[Using addition to explain multiplication video](https://youtu.be/LSJ9WcPuhD4) | **Lattice Multiplication**National Center for Intensive Intervention [Lattice Multiplication](https://www.youtube.com/watch?v=W7cTvEcyyj4) | **Comparison*** Use a number line.
* Show zero
* Show infinite (- & +)

Khan Academy [Multiplication on a Number line](https://youtu.be/PLDfl6daajo) | **Area Model**National Center for Intensive Intervention[Area Model](https://youtu.be/NbheCaFQ-cI) |

#### Division

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| **Standard Division**Khan Academy [Introduction to Long Division](https://www.khanacademy.org/math/arithmetic-home/multiply-divide/mult-digit-div-2/v/division-2) | **Partial Quotients**Khan Academy [Idea of Division](https://www.khanacademy.org/math/arithmetic-home/multiply-divide/division-intro/v/the-idea-of-division)National Center on Intensive Interventions [Base 10 Blocks Division Video](https://youtu.be/6mhbMLV0-yE) National Center on Intensive Interventions [Basic Facts Dividing Video](https://youtu.be/1jY4dcj6WGY)National Center on Intensive Interventions [Partial Quotients Video](https://youtu.be/RewZrJgX8kU) | **Lattice Division**National Center on Intensive Intervention[Lattice Division Video](https://www.youtube.com/watch?v=P4b64TjxF3g&list=PL1bOssvlvTAVJkNyW6W4DS6VQHT8Asiaa) |

##### Additional Resources

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| [Intensive Interventions and Lesson Plans](https://intensiveintervention.org/implementation-intervention/math-lessons)The National Center on Intensive Intervention identified several strategies and interventions that enhance the skills of students with math difficulties in number system and counting, place value computation, basic facts, fractions as numbers, place value concepts, and computation of fractions.The IRIS Center. (2017). High-quality mathematics instruction: What teachers should know. Retrieved from <https://iris.peabody.vanderbilt.edu/module/math/>[Learning Mathematics through Representations](https://sites.google.com/view/lmrberkeleyedu)“Learning Mathematics through Representations (LMR) is a research-based curriculum unit for the teaching and learning of integers and fractions in the elementary grades (26 lessons). **Henrico County, VA** [Mathematics Courses – Activities and links that are aligned to grade level SOL](https://sites.google.com/henrico.k12.va.us/hcpsmathematics/courses?authuser=0)**University of Texas** Researchers at the University of Texascreated Instructional Routines for Mathematics Intervention documents that have pre-created resources and materials for mathematics for 23 interventions, which focus on different mathematical content. Each of the 23 interventions include vocabulary cards and problem sets to use during instruction. The interventions all require explicit instruction. Though the interventions align with standards from the Texas Essential Knowledge and Skills (TEKS), the resources and materials apply to teaching mathematics skills identified in the Virginia Standards of Learning. * [Instructional Routines for Mathematics Intervention User Guide (31 pages)](https://4.files.edl.io/4749/04/23/21/225643-19d9c345-7899-42c2-a04a-85319467e96e.pdf)
* [All of the Instructional Routines Files (2951 pages)](https://4.files.edl.io/797d/04/23/21/225638-0e72f842-7c82-4a19-be6c-28551e785665.pdf)
* [Texas Instructional Routines for Mathematics Interventions Modules](https://www.inclusionintexas.org/apps/pages/index.jsp?uREC_ID=2155039&type=d&pREC_ID=2169859)
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##### References

The IRIS Center. (2015). Intensive intervention (part 2): Collecting and analyzing data for data-based individualization. Retrieved from <https://iris.peabody.vanderbilt.edu/module/dbi2/>

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